



ADVANCED RESEARCH CENTER ON ELECTRONIC SYSTEMS
FOR INFORMATION AND COMMUNICATION TECHNOLOGIES
“ERCOLE DE CASTRO” (ARCES)

Short course for the PhD Program in Electronics,
Telecommunications and Information Technology Engineering
(ET-IT) at the University of Bologna

Digital High-Order $\Delta\Sigma$ Modulators: the Design of Noise Shaping Features as an Optimization Problem

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Duration: 8 hours distributed through 4 lectures
Collocation: 22-23 July 2015
Language: English
Technical areas: Circuits and Systems, Digital Signal Processing, Nonlinear systems, Convex Optimization

Abstract

$\Delta\Sigma$ modulation is best known for oversampling A/D conversion. However, its most frequent application is as a re-coding approach in the digital realm. Applications range from D/A conversion to “digital” audio amplification, from frequency-synthesis to the storage of analog waveforms in formats that simplify their reconstruction. Working in the numeric domain offers levels of flexibility unknown in analog modulators, enabling the implementation of high-order structures, an accurate tuning of their noise-shaping properties, and a full adaptation to the systems where they are embedded. Yet, this potential is often underemployed by many common design flows. This short course illustrates advanced techniques and optimization methods for the design of the noise shaping properties, which allow them to be tuned to suit specific application needs. Theory is coupled to practical examples ranging from audio coders capable of distributing noise in a psychoacoustically optimal way, to fractional-PLLs, to the more exotic use of $\Delta\Sigma$ modulators as solvers for certain classes of optimization problems. As a bonus, an open source toolbox based on the Python programming language is offered to the participants to experiment with $\Delta\Sigma$ modulators and their noise transfer functions. The toolbox is used throughout the short course to demonstrate the proposed concepts and the course is also an occasion to get introduced to scientific Python.

1 Learning objectives

Those who attend the short course will:

- review or become familiar with the basic operation of a digital $\Delta\Sigma$ modulator, recognizing the differences between analog and digital modulators and understanding how the noise shaping features can be tuned;
- review or become familiar with some major applications of digital $\Delta\Sigma$ modulators, such as in audio coding, digital amplification, digital to analog conversion, frequency synthesis by *fractional* phase locked loops (PLLs);
- find out about the major assumption that are made in the design of the noise shaping feature of $\Delta\Sigma$ modulators, recognizing their limitations and implications;
- learn the inner working of some standard procedures for the design of high-order single-quantizer modulators (as they are found in the Literature and in some CAD tools) and see how they can be modified and augmented to suit specific application needs;
- understand how the design of the modulator noise shaping features should go beyond the conventional distinction between a signal band and the rest of the available frequency range, since different application environments can perceived noise in different ways;
- learn how to formalize the actual application needs with respect to the spectral shaping of quantization noise through a *filtered approximation* paradigm;
- find out how this paradigm can be applied to deploy formal *convex optimization* techniques in the fine tuning of the modulator design;
- learn how to apply these concepts to the design of modulators for specific tasks including: audio coders capable to distribute quantization noise in a psychoacoustically optimal way; switched mode drivers adapted to the actual ability of the load to practice noise removal; fractional PLLs where the impact of the $\Delta\Sigma$ modulator quantization noise over the output phase noise is minimized;
- gather information on what can go wrong (namely turn out to be different from the models used in design) in the actual implementation of a digital $\Delta\Sigma$ modulator;
- understand how the task performed by a high order $\Delta\Sigma$ modulator is in many ways itself assimilable to that of an optimizer.

In the process, the audience will be exposed to advanced concepts, such as the reformulation of design constraints via the Kalman–Yakubovich–Popov lemma and the formalization of the modulator design as a semidefinite-program expressible by linear matrix inequalities and solvable by interior point methods.

At the end of the short course, the audience will see how the proposed concepts can be put together and exploited also for rather unusual tasks. Specifically, the design of very specialized modulators that can be used as solvers for restricted classes of integer optimization problems will be considered and exemplified. Even if such an application currently has only limited practical value, it is in many senses representative of the actual modulator operation and of level of tuning that can be achieved. With it, the audience will gain sensitivity on the potential of $\Delta\Sigma$ modulation and on non-classical interpretations of the modulators' behavior.

The short course will join the delivery of theoretical information with practical competence. This will be achieved by coupling lecturing with the practical demonstration of *methods* and *algorithms*. Demos originating from real world problems will be proposed by running design and simulation tools on a computer connected to a screen projector. For instance, in order to show how the noise shaping features should be tied to the actual noise perception in the application environment, the design of a modulator for audio applications will be exemplified, using a psycho-acoustic model as the basis for its design. Simulation will also be used to point out phenomena such as the production of undesired tones and how the dosing of random dithering can be used to attenuate this issue. The demos will be coded and run using a *scientific Python* programming environment.

With the points above, the attendants shall also get the following side benefits:

- enhance their perception of how design activity in the electronics, telecommunication and information technology field is increasingly becoming an interdisciplinary playground: in the proposed examples electronics, digital signal processing, optimization and computer based design aids are tightly intermixed;
- see how the formalization of design problems as optimization problems let one develop tools favoring a *declarative style* where *specifications* (a description of the requirements of the embedding environment) rather than *instructions* (a list of elementary operations to be practiced in sequence) are put at the center of the design activity;
- become familiar the *scientific Python* programming environment. Python is a very modern programming language that together with packages that providing efficient matrix manipulation, numerical and symbolic differentiation, numeric algorithms for a variety of tasks, and powerful plotting abilities becomes an extremely serious competitor to traditional numerical computation environments such as Matlab, Scilab or Octave. Scientific Python is currently the object of courses at many institutions including Cineca.

As a bonus, the audience will be offered a software toolbox for experimenting with $\Delta\Sigma$ modulators and their noise transfer function. The package, named PyDSM, is coded in Python and is independent from conventional numerical computation environments. It is open-source and only requires free software to run, including optimization libraries recently developed in academic institutions. Furthermore, it can be used on all major operating systems (Windows, Linux, MacOS). PyDSM will be introduced during the short course and then used for most of the demos and examples. Obviously, the goals and duration of the course will not

enable a thorough illustration of all its features, yet the audience will receive sufficient information to jump over the first part of its learning curve. The toolbox is expected to help the audience experiment with the proposed concepts after the course and possibly apply them in research and education. In fact, by its open nature it can easily accommodate new scientific results.

2 Target audience and prerequisite knowledge

The short course is aimed at PhD students in the electronics, telecommunications and information technology area and organized so that attendance is possible whatever the year the students are attending through their PhD program.

The following background knowledge is expected from the participants:

Digital signal processing. Basic knowledge required, including the ability to design fundamental filter structures, use the Laplace transform, evaluate filter features. The ability to manage concepts such as deterministic and random signals and their characterization is also expected.

Linear system theory. Basic knowledge required, including the ability to manage concepts such as feedback, stability, passivity.

Mathematics. The ability to work comfortably with linear algebra and expressions in matrix form is required. The concept of convexity, related to functions and sets is necessary for the parts of the course where optimization techniques are introduced.

Additionally, the following competences are not strictly required, but can be an advantage:

- Applied electronics. As most examples originate from real world problems, some knowledge of the involved circuits and electronic architectures can be useful. This includes basic knowledge of switched mode amplification via power bridges.
- Programming. The course includes demos based on a design toolbox. The ability to use numerical computation environment (e.g., Matlab, Octave, Scilab) or the knowledge of a programming language can help appreciating the toolbox utilization.
- Optimization. Some of the proposed design techniques rely on advanced optimization techniques for convex problems. Furthermore, during the course a parallel is made between the operation of a $\Delta\Sigma$ modulator and that of an optimizer for unconstrained integer quadratic programs. Consequently, some elementary knowledge about optimization methods may be beneficial.

In any case, the short course is structured to keep hard dependencies on other subjects to a minimum and to be as much as possible self-contained, in order to make the discussion accessible to the widest possible audience.

3 Course schedule

The short course is organized in 4 units, each approximately 2 hours long (two 50' modules plus a break). This corresponds to four main parts, where the audience is brought from the fundamental operation of the modulators to the illustration of techniques for the design of the noise shape properties and to the management of deviations from the expected behaviors. These parts are characterized by increasing degrees of sophistication and include many examples tackling real world problems.

Each part comprises a *hands-on* section where more practical or speculative concepts are presented. For instance, there is: an introduction to scientific Python; a presentation of the PyDSM open source toolkit for experimenting with modulators and their noise transfer functions; applicative examples; and a *finale* where all the previously discussed ideas are put together in order to use the modulator as an heuristic solver for certain classes of optimization problems. This application currently only bears limited practical value, yet is in many sense representative of the actual operation of the modulator and of the level of tuning and specialization that can be achieved.

3.1 The software toolbox used for demoing



PyDSM is an open source code including a fast time-domain simulator for $\Delta\Sigma$ modulators, as well as all the most important algorithms for the design of their noise transfer functions.

Specifically, all the methods that are customarily included in packages that are *de-facto* standards (such as, for instance, Richard Schreier's DELSIG for MATLAB) are also included in PyDSM. In addition to them, PyDSM incorporates many advanced strategies (e.g., some design techniques based on semidefinite programming) and some specialized ones (e.g., Dunn and Sandler's approach to designing psychoacoustically optimal modulators). What is notable is that all these strategies were never collected into a single toolbox before. Note that PyDSM is currently under active development, so that it is impossible to exhaustively list in this proposal all the abilities that it will have by the time the course is delivered.

A key peculiarity of the PyDSM toolbox is that in addition to being open-source, it only needs free software as its pre-requisites. Thanks to this property, it can be tested and deployed without worrying about licensing issues and without the need to acquire expensive packages or frameworks. A Windows, Linux or Mac computer with the bare OS is all that is needed to experiment. This is an appealing property, that can greatly simplify the deployment of the code in resource constrained environments, such as start-ups, institutions in development countries, and particularly in education. PyDSM can be freely passed to students that can bring it home, share it with colleagues, modify it, without the need to acquire an expensive running environment, or the temptation to break copyright law.

PyDSM is coded in Python, with the help of standard packages for scientific computing, such as Numpy, Scipy, and Matplotlib. It also takes advantage of some state of the art libraries from the academia, including CVXOPT from UCLA (an advanced convex optimization backend) and CvxPy from Stanford (a package for modeling convex optimization programs via the Disciplined Convex Programming approach).

PyDSM can be used in conjunction with a working interface that is extremely configurable and that can be made similar to conventional numerical computation environments like Matlab or Scilab. This can streamline the process of getting accustomed to it. Similarly to Matlab and Scilab, the environment can offer powerful plotting abilities that are quite useful in signal processing. At the same time, the toolbox is based on a real and full-fledged programming language. This means that PyDSM can also be run without a 'front-end', used to code stand-alone programs, or embedded in larger projects. The ability to count on a modern programming language that supports multiple programming paradigms, and is designed for interactivity, conciseness and rapid development may represent a fresh take with respect to conventional scientific environments.

The toolbox can be downloaded at pydsm.googlecode.com.

4 Course authors and instructors

Instructor



Sergio Callegari is with the Advanced Research Center on Electronic Systems for Information and Communication Technologies “E. De Castro” (ARCES) and with the Department of Electrical, Electronic and Information Engineering “Guglielmo Marconi” (DEI) at the University of Bologna, Italy.

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Sergio Callegari received a Dr. Eng. degree (with honors) in electronic engineering and a Ph.D. degree in electronic engineering and computer science from the University of Bologna, Italy, in 1996 and 2000, respectively, working on the study of nonlinear circuits and chaotic systems. In 1996, he was a visiting student at King’s College London, UK.

He is currently a researcher and assistant professor at the School of Engineering II, University of Bologna, where he teaches “Electronics for analog signal processing” and “Applied Analog Electronics and Sensing” to students of Electronic Engineering and Aerospace Engineering. He is also a faculty member of the Advanced Research Center on Electronic Systems (ARCES) at the University of Bologna.

In 2008, 2009, 2011 he was a visiting researcher at the University of Washington in Seattle for short periods. His current research interests include nonlinear signal processing, internally nonlinear, externally linear networks, chaotic maps, delta-sigma modulation, testing of analog circuits, and random number generation.

He has authored or co-authored more than 80 papers in international conferences, journals and scientific books, as well as 4 national patents. In 2004 he was co-recipient of the IEEE Circuit and Systems Society Darlington Award, for the best paper appeared in the IEEE Transactions on Circuits and Systems in the previous biennium.

He is an IEEE Senior Member and he served as an Associate Editor for the IEEE Transactions on Circuits and Systems – Part II during 2006–2007 and as an Associate Editor for the IEEE Transactions on Circuits and Systems – Part I during 2008–2009. He is currently serving as an Associate Editor for the IEICE Nonlinear Theory and its Applications (NOLTA) Journal. For the same journal, he has already served as guest associate editor for a few special issues. He is Secretary of the Technical Committee on Nonlinear Circuits and Systems and also member of the Technical Committee on Education and Outreach in the IEEE CAS Society. He also served in the Organization Committee and as Publication Co-Chair at NOLTA 2006 (Bologna), as a member of the Organization Committee of Eurodoc 2006 (Bologna), as a Special-Session Co-Chair of NOLTA 2010 (Krakow), as a Co-Chair for the Nonlinear Circuits and Systems track at ICECS 2012 and 2013, and as a chair for Nonlinear Circuits and Systems track at ISCAS 2013.

Recent publications related to the course

- [SC-1] Sergio Callegari. *Coding of stereo signals by a single digital $\Delta\Sigma$ modulator*, in Proceedings of IEEE ICECS 2013, Abu Dhabi (UAE), Dec. 2013, pp. 589–592.
- [SC-2] Sergio Callegari, Federico Bizzarri, Riccardo Rovatti, Gianluca Setti. *On the Approximate Solution of a Class of Large Discrete Quadratic Programming Problems by $\Delta\Sigma$ Modulation: the Case of Circulant Quadratic Forms*. IEEE Transactions on Signal Processing, Vol. 58, N. 12, pages 6126–6139. 2010.
- [SC-3] Sergio Callegari, Federico Bizzarri. *Output Filter Aware Optimization of the Noise Shaping Properties of $\Delta\Sigma$ Modulators via Semi-Definite Programming*, IEEE Transactions on Circuits and systems – Part I: Regular Papers, Vol. 60, N. 9 Pages 2352–2365, 2013.
- [SC-4] Sergio Callegari, Federico Bizzarri. *Should $\Delta\Sigma$ modulators used in AC motor drives be adapted to the mechanical load of the motor?*, Proceedings of the 19th IEEE International Conference on Electronics, Circuits and Systems (ICECS), 2012, pp. 849–852.
- [SC-5] Federico Bizzarri, Sergio Callegari, Giambattista Gruosso. *Towards a nearly optimal synthesis of power bridge commands in the driving of AC motors*, IEEE International Symposium on Circuits and Systems (ISCAS) 2012, Pages 2119–2122.

- [SC-6] Federico Bizzarri, Christoph Buchheim, Sergio Callegari, Alberto Caprara, Andrea Lodi, Riccardo Rovatti, Gianluca Setti. *Practical Solution of Periodic Filtered Approximation as a Convex Quadratic Integer Program*. Proceedings of the First International Conference on Complex System Design & Management CSDM 2010. Pages 149–160.
- [SC-7] Sergio Callegari, Federico Bizzarri. *A Heuristic Solution to the Optimisation of Flutter Control in Compression Systems (and to Some More Binary Quadratic Programming Problems) via $\Delta\Sigma$ Modulation Circuits*. Proceedings of 2010 IEEE International Symposium on Circuits and Systems (ISCAS) 2010, Pages 1815–1818.
- [SC-8] Federico Bizzarri, Sergio Callegari, Riccardo Rovatti, Gianluca Setti. *On the synthesis of periodic signals by discrete pulse-trains and optimisation techniques*. European Conference on Circuit Theory and Design (ECCTD) 2009. Pages 584–587.
- [SC-9] Sergio Callegari, Federico Bizzarri. *Noise Weighting in the Design of $\Delta\Sigma$ Modulators (with a Psychoacoustic Coder as an Example)*, IEEE Transactions on Circuits and systems — Part II: Express Briefs, Vol. 60, N. 11 Pages 756–760, 2013.
- [SC-10] Sergio Callegari, Federico Bizzarri. *Optimal design of the noise transfer function of $\Delta\Sigma$ modulators: IIR strategies, FIR strategies, FIR strategies with preassigned poles*, Signal Processing, Elsevier, in press.
- [SC-11] Sergio Callegari, Federico Bizzarri. *Optimal Quantization Noise Management in Wide-band Fractional-N PLLs*, Proceedings of 2015 IEEE International Symposium on Circuits and Systems (ISCAS) 2015, in press.

Co-authors of part of the material



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Federico Bizzarri was born in Genoa, Italy, in 1974. He received the Laurea (M.Sc.) five-year degree (summa cum laude) in electronic engineering and the Ph.D. degree in electrical engineering from the University of Genoa, Genoa, Italy, in 1998 and 2001, respectively.

From June 2010 to June 2013, he has been a temporary research contract Assistant Professor at the Dipartimento di Elettronica, Informazione e Bioingegneria of the Politecnico di Milano, Milan, Italy, where he is currently a temporary research fellow. In 2000, he was a visitor to EPFL, Lausanne, Switzerland. From 2002 to 2008 he had been a post-doctoral research assistant in the Biophysical and Electronic Engineering Department of the University

of Genova, Italy. In 2009 he was a post-doctoral research assistant in the ARCES Research Center of the University of Bologna, Italy, where he is a research fellow till April 2013.

His main research interests are in the area of nonlinear circuits, with emphasis on chaotic dynamics and bifurcation theory, circuit models of nonlinear systems, image processing, circuit theory and simulation. He is the author or coauthor of about 60 scientific papers, more than an half of which have been published in international journals.

He is an IEEE Senior Member. Since December 2012 he has been serving as Associate Editor for the IEEE Transactions on Circuits and Systems – Part I. He was the co-recipient of the Best Associate Editor Award 2013 – IEEE Transactions on Circuits and Systems I. He was also a member of the Review Committee for ISCAS 2013.



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Michael Peter Kennedy is Professor of Microelectronic Engineering at University College Cork (UCC). He received the BE (Electronics) degree from UCD in 1984, the MS and PhD from the University of California at Berkeley in 1987 and 1991, respectively, and the DEng from the National University of Ireland in 2010. He joined UCC as Chair of the Department of Microelectronic Engineering in 2000. He served as Dean of the Faculty of Engineering from 2003 through 2005 and as Vice-President for Research from 2005 to 2011. He has over 330 research publications (including four patents) in the fields of oscillator design, hysteresis, neural networks, nonlinear dynamics, chaos communication, mixed-signal test, and frequency synthesis. He has worked as a consultant for SMEs and multinationals in the microelectronics industry and is founding Director of the Microelectronics Industry Design Association (MIDAS Ireland) and the Microelectronics Competence Centre of Ireland (MCCI).

He was made a Fellow of the Institute of Electrical and Electronic Engineers (IEEE) in 1998 for contributions to the theory of neural networks and nonlinear dynamics and for leadership in nonlinear circuits research and education. He has received many prestigious awards including Best Paper (International Journal of Circuit Theory and Applications), the 88th IEE Kelvin Lecture, IEEE Millenium and Golden Jubilee Medals, the inaugural Royal Irish Academy Parsons Award in Engineering Sciences, and the IEEE Solid-State Circuits Society Chapter of the Year Award 2010. In 2004, he was elected to membership of the Royal Irish Academy and was made a Fellow of the Institution of Engineers of Ireland by Presidential Invitation. From 2005 to 2007, he was President of the European Circuits Society and Vice-President of the IEEE Circuits and Systems (CAS) Society (with responsibility for Europe, Africa and the Middle East).

In 2012, he was appointed as a Distinguished Lecturer of the IEEE CAS Society and was elected Secretary for International Relations of the Royal Irish Academy.

Recent publications related to the course

- [MPK-1] Z. Ye and M.P. Kennedy. *Reduced Complexity MASH Delta - Sigma Modulator*. IEEE Trans. Circuits and Systems — Part II, 54(8):725 - 729, Aug. 2007.
- [MPK-2] K. Hosseini and M.P. Kennedy. *Maximum Sequence Length MASH Digital Delta-Sigma Modulators*. IEEE Trans. Circuits and Systems — Part I, 54(12):2628 - 2638, Dec. 2007 (MIDAS Ireland Outstanding PhD Research Publication 2008).
- [MPK-3] Z. Ye and M.P. Kennedy. *Hardware Reduction in Digital Delta-Sigma Modulators via Error Masking — Part I: MASH DDSM*. IEEE Trans. Circuits and Systems — Part I , 56(4):714 - 726, Apr. 2009.
- [MPK-4] Z. Ye and M.P. Kennedy. *Hardware Reduction in Digital Delta-Sigma Modulators via Error Masking — Part II: SQ-DDSM*. IEEE Trans. Circuits and Systems — Part II , 56(2):112 - 116, Feb. 2009.
- [MPK-5] K. Hosseini, M.P. Kennedy, S.H. Lewis and B.C. Levy. *Prediction of the Spectrum of a Digital Delta-Sigma Modulator Followed by a Polynomial Nonlinearity*. IEEE Trans. Circuits and Systems — Part I , 57(8):1905 - 1913, Aug. 2010.
- [MPK-6] K. Hosseini, M.P. Kennedy. *Minimizing Spurious Tones in Digital Delta-Sigma Modulators*. Springer, 2011. 148pp.
- [MPK-7] B. Fitzgibbon, M.P. Kennedy, F. Maloberti. *Hardware Reduction in Digital Delta-Sigma Modulators via Bus-Splitting and Error Masking — Part I: Constant Input*. IEEE Trans. Circuits and Systems — Part I , 58(9):2137 - 2148, Sept. 2011.
- [MPK-8] B. Fitzgibbon, S. Pamarti and M.P. Kennedy. *A Spur - Free MASH DDSM with High-Order Filtered Dither*. IEEE Trans. Circuits and Systems — Part II , 58(9):585 - 588, Sept. 2011.
- [MPK-9] K. Hosseini, B. Fitzgibbon and M.P. Kennedy. *Observations Concerning the Generation of Spurious Tones in Digital Delta-Sigma Modulators Followed by a Memoryless Nonlinearity*. IEEE Trans. Circuits and Systems — Part II , 58(11):714 - 718, Nov. 2011.
- [MPK-10] M.P. Kennedy. *Nested digital delta-sigma modulator*, U.S. Provisional Appln. No. 61/576411, filed 16 December 2011.
- [MPK-11] M.P. Kennedy, B. Fitzgibbon. *Pipelined Bus - Splitting Digital Delta-Sigma Modulator for Fractional-N Frequency Synthesizer System and Method*, U.S. Provisional Appln. No. 61/653754, filed 31 May 2012.
- [MPK-12] B. Fitzgibbon, M.P. Kennedy and F. Maloberti. *Hardware Reduction in Digital Delta-Sigma Modulators via Bus-Splitting and Error Masking — Part II: Non-constant Input*. IEEE Trans. Circuits and Systems — Part I, 59(9):1980 - 1991, Sep. 2012.

[MPK-13] M.P. Kennedy. *Recent Advances in the Design, Analysis and Optimization of Digital Delta-Sigma Modulators*. Trans. IEICE, Volume 3, Issue 3, pp. 258-286 (2012).